Name _____

Work for entering AP Calculus AB

Summer Work 2023

This workbook contains problems designed to ensure the student's readiness for AP Calculus AB. The ten topics covered in this packet are concepts that should be mastered before entering AP Calculus AB. Assistance with the packet will be provided at the beginning of the school year. Completion of this packet over the summer before beginning AP Calculus AB will be of great value to helping students successfully meet the academic challenges awaiting them in AP Calculus AB and beyond.

Topics Covered in this Packet:

1. Functions (evaluation, properties, types, transformations, notation)
2. Trigonometry
3. Solving Equations
4. Evaluating and order of operations
5. Exponentials and Logarithms
6. Quadratics
7. Linear equations
8. Interval notation
9. Rational Expressions and Equations
10. Radical Expressions and Equations

Instructions:

- Start review packet about 2 weeks prior to the start of the school year.
- You are expected to use your notes from previous courses.
- Show all work (when appropriate) on a separate sheet of paper
- Simplification is implied for all problems.
- Leave answers in fractional form when appropriate (improper fractions are preferred to mixed numbers).
- Problems are to be returned on the first full Friday of classes (which is NOT the first day of school). Students will have the chance to ask questions before submitting the work. A test reviewing the prerequisite skills (reviewed in this packet) will be conducted the following week of classes.

Remember: This packet is for your benefit and is intended to help you succeed as you move through more advanced math classes.

NO CALCULATORS ALLOWED

Section A

Be able to complete a multiplication table similar to the one shown below in 2 minutes 30 seconds or less

	1	6	9	5	2	7	4	3	8
2									
4									
1									
6									
3									
8									
5									
7									
9									

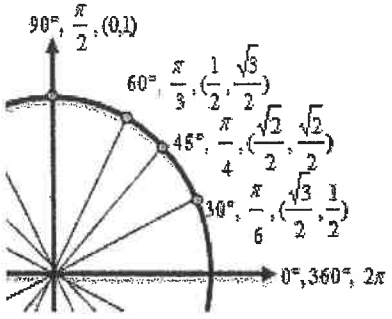
Time taken: _____

Be able to complete a trig values table similar to the one shown below in 2 minutes 30 seconds or less

	$\sin x$	$\tan x$	$\cot x$	$\cos x$	$\sec x$	$\csc x$
0						
$\frac{\pi}{3}$						
$\frac{\pi}{2}$						
$\frac{\pi}{6}$						
$\frac{\pi}{4}$						

Section B

UNIT CIRCLE



You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as sin/cos or the slope of the line.

Examples:

$\sin\frac{\pi}{2}=1$ $\cos\frac{\pi}{2}=0$ $\tan\frac{\pi}{2}=und$

***You must have these memorized OR know how to calculate their values without the use of a calculator.**

a.) $\sin\pi$ *b.)* $\cos\frac{3\pi}{2}$ *c.)* $\sin\left(-\frac{\pi}{2}\right)$ *d.)* $\sin\left(\frac{5\pi}{4}\right)$

e.) $\cos\frac{\pi}{4}$ *f.)* $\cos(-\pi)$ g) $\cos\frac{\pi}{3}$ h) $\sin\frac{5\pi}{6}$

i) $\cos\frac{2\pi}{3}$ j) $\tan\frac{\pi}{4}$ k) $\tan\pi$ l) $\tan\frac{\pi}{3}$

m) $\cos\frac{4\pi}{3}$ n) $\sin\frac{11\pi}{6}$ o) $\tan\frac{7\pi}{4}$ p) $\sin\left(-\frac{\pi}{6}\right)$

1.

2. **Find the solution(s) to the equations on [0,2π].**
   a. $2\sin^2\theta = 1 - \sin\theta$
   b. $2\tan\theta - \sec^2\theta = 0$
   c. $\cos^2(\theta) + \sin\theta = 1$

3. Circle the two expressions that are equivalent.

a) $\cos^2(x)$ b) $(\cos x)^2$ c) $\cos x^2$

4. Circle the two expressions that are equivalent.

a) $(\sin x)^{-1}$ b) $\arcsin(x)$ c) $\sin x^{-1}$ d) $1/\sin(x)$

5. Sketch a reasonable graph of $sin^{-1}(x)$

Section C

EVEN AND ODD FUNCTIONS

Recall:

Even functions *are functions that are symmetric over the y-axis.*

To determine algebraically we find out if $f(x) = f(-x)$

*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected across the y-axis*)*

Odd functions *are functions that are symmetric about the origin.*

To determind algebraically we find out if $f(-x) = -f(x)$

*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected over the origin*)*

1. Determine if the function is even, odd or neither. Show your work.

a. $y = 5 - \sqrt{9 - x^2}$

b. $f(x) = x - \frac{1}{x}$

c. $5x^2 - 6y = 1$

d. $y = e^x - \frac{1}{e^x}$

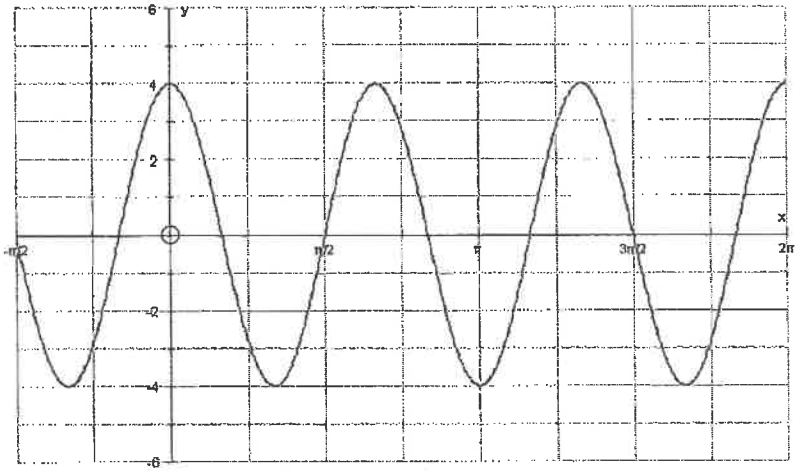
e. $g(x) = x^5 - 3x^3 + x$

f. $j(x) = 2\cos x$

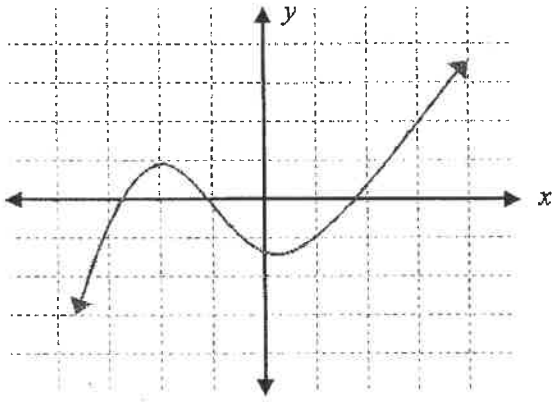
g. $k(x) = \sin x + 4$

h. $r(x) = \frac{x^2 + 1}{2x^3 - x}$

i.



j.



2. Find $f(x+h)$ for $f(x) = x^2 - 2x - 3$.

3. Simplify:

a. $$\frac{(x+1)^3(4x-9)-(16x+9)(x+1)^2}{(x-6)(x+1)^3}$$

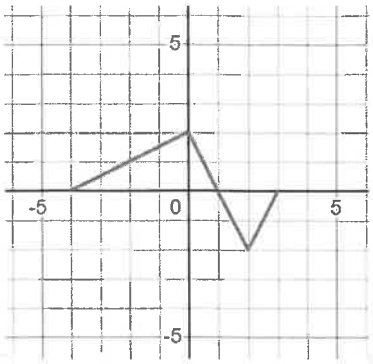
b. $$\frac{\sqrt{x-2}+\dfrac{5}{\sqrt{x-2}}}{x-2}$$

4. Find all points of intersection of the graphs of $x^2 + 3x - y = 3$ and $x + y = 2$.

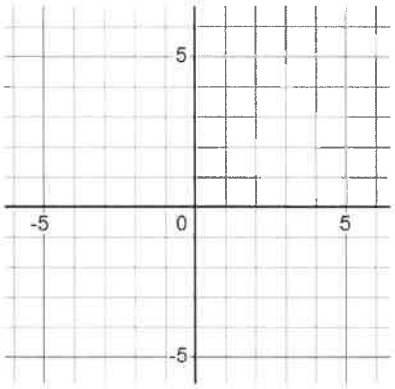
5. Find the area of the region bounded by $y = \sqrt{36 - x^2}$ and the x-axis.

6. Solve $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

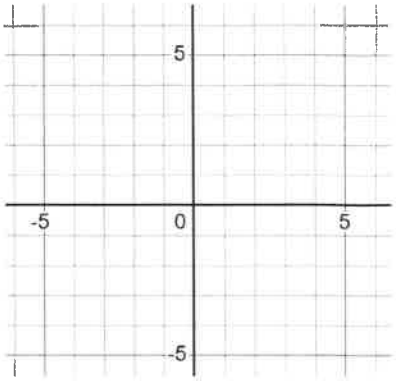
7. Here is the graph of $f(x)$. Use it to graph the below relationships.



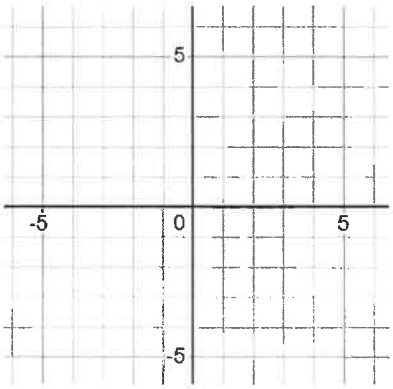
a. $2f(x)$



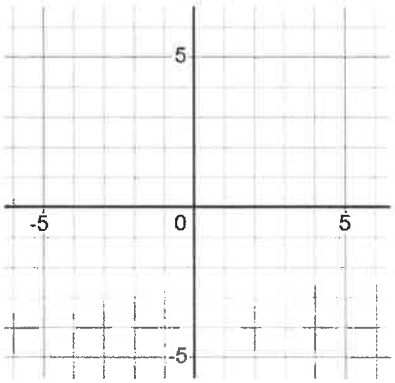
b. $f(x-1)$



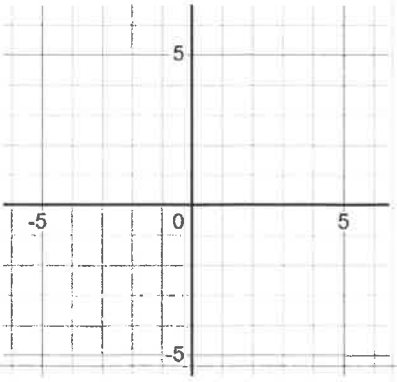
c. $|f(x)|$



d. $f(|x|)$



e. $f^{-1}(x)$



Section D

1. Write in interval notation
   a. $x > -6$
   b. $-9 \le x < 2$
2. Determine the equations of the following lines. Give your answer in point-slope form.
   a. the line through (-2,5) and (3,-4);
   b. the line through (0, 3) and the midpoint of the line segment from (-1, 4) to (3, 2).
3. Is $y = \sqrt{5x-12}$ a function?
4. Is $x = y^4$ a function?
5. Which quadrant is the point (-31,-2) in?
6. Let $f(x) = 3x^2 + 2x - 1$. Find
   a. $f(-1) =$
   b. $f(-x) =$
7. Suppose the point (-7,2) is on the graph of $y = f(x)$. Find a point on the graph of
   a. $y = f(x+1)$
   b. $y = f(x) - 3$

Section E

Evaluate

1. $2^3 \cdot 5 \cdot 8^0 =$	2. $[15(10) - 12(10)] \div 10$	3. $\frac{9}{2} \div \left(2\frac{3}{8}\right)$
4. $(-1)^{2022}$	5. $6\lvert 8 - 10 \rvert$	6. $24 \div 3 \cdot 2 - 5 \cdot 7$
7. -1^{2022}	8. $4[(3+2\times 3)-5]+7$	9. $3(1-8)-6+7$
10. 3^4	11. $\dfrac{\frac{3}{2}+\frac{3}{4}+\frac{3}{8}}{21}$	12. $\left(\dfrac{6+(-9)}{-3-9}\right)\left(\dfrac{24+(-72)}{3-6}\right)$
	13. $322 \div 14 + 20 \div 4 \cdot 5$	

Simplify:

14. $\frac{12}{54}$	15. $1\frac{2}{3}+4\frac{1}{5}-3\frac{5}{6}$	16. $\frac{4}{5}\div\frac{2}{15}$
17. $\left(\frac{5u^2}{2v^2}\right)^2$	18. $\frac{2x^{-2}y}{3y^{-3}x^2}\cdot\frac{3x^4}{8y^{-2}}$	19. $\left(3^{-1}+2^{-1}\right)^2$
20. $(10z)(11^0)$	21. $3\sqrt{8}-3\sqrt{32}$	22. $\sqrt{17}+5\sqrt{17}$
23. $\frac{2}{\sqrt{12}}$	24. $\sqrt{5}\cdot\sqrt{15}$	25. $\sqrt{2}\left(3\sqrt{2}+5\right)$
26. $\frac{1}{3-\sqrt{2}}$	27. $(7x-2y)-(3x+5y)$	28. $(2x-9)(3x-8)$
29. $(8+x)(8-x)$	30. $(5x+2y)^2$	31. $\frac{3x}{6x^2+3x}$
32. $\frac{x^2-2x-8}{x^2-16}$	33. $\frac{\frac{1}{3}-\frac{1}{x}}{\frac{1}{9}-\frac{1}{x^2}}$	34. $\frac{1}{1+\frac{2}{3-\frac{1}{2}}}$
35. i^3	36. i^{100}	

Graph:

37. $y=-2x+2$	38. $y>x-4$	39. $y=x^2$
40. $y=\frac{1}{-2x+2}$	41. $x-2y>4$	42. $y=(x-3)^2$
43. $x^2+y^2=25$	44. $\frac{x^2}{36}+\frac{y^2}{4}=1$	45. $x=-2$

Factor:

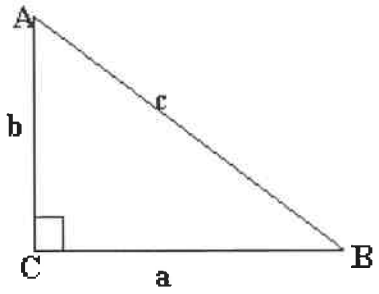
46. x^2-4x	47. a^2-16	48. x^2-4x+4
49. s^2+5s+4	50. $x^2-9x-10$	51. $x^2-10x-24$
52. $x^2-10x+24$	53. $x^2-10x-25$	54. $2x^2-3x-9$
55. $4x^2+12x+9$	56. $8x^2-14x+5$	57. $2y^2+10y-12$
58. x^2+64	59. x^2-81	60. $3x^3-6x^2-4x+8$
61. x^4-1	62. $4x^3-8x^2-25x+50$	

63. Is -4 a root of $P(x)=x^3+6x^2-x-30$?

64. The vertex of the graph of $y=(x+3)^2+2$ is at _____

65. Given the triangle shown, find:

66.



a. $\sin A=$

b. $\cos B=$

c. $\tan A=$

d. $\sec A=$

e. $\cot B=$

67. Find the slope of the line passing through the points $(-4,5)(-3,8)$

68. Find the slope and y-intercept of the line $y=\frac{5}{6}x-1$

69. Write the equation of the line in slope-intercept form of the line that contains the point $(8,-6)$ and has slope of $\frac{-3}{4}$

70. Which is larger, 2^5 or 5^2

71. Evaluate: $\ln e^{14}$

72. Evaluate: $10^{\log 7}$

73. Evaluate: $\sum_{n=1}^{3} 2n-2$

74. Divide: $(x^3+3x^2+3x+1)\div(x+1)$

75. Solve the equation for x: $(7^3)^5 = 7^x$

76. Rewrite the expression using positive exponents. $(-2x)^{-3}$

Let $f(x)=2x+3$ and $g(x)=x^2$

77. Evaluate $g(-3)$	78. Evaluate $f(1)$	79. Evaluate $f(g(10))$

Solve:

80. $4-2(y+6)=11-3y$	81. $\frac{x}{2}+4=3x$	82. $\lvert x+1\rvert=5$
83. $x^2=16$	84. $\frac{1}{3}+\frac{1}{x}=\frac{1}{2}$	$x+y=6$ 85. $3x-y=30$
86. $\frac{x}{4x-8}=\frac{2}{x}$	87. $\sqrt{x}+5=3$	88. $2=\sqrt{3x+1}-3$
89. $x^2-6x+4=20$	90. $\frac{10}{x-2}+3=\frac{5x}{x-2}$	

91. Solve by completing the square: $x^2+10x=6$

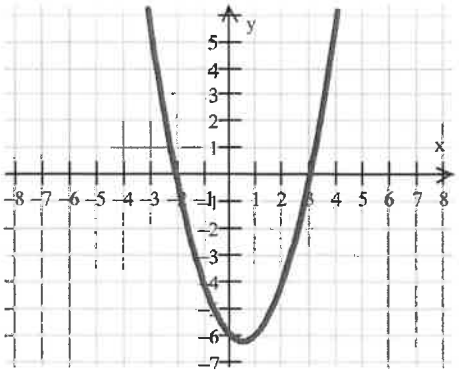
92. Use the quadratic formula to solve the equation $0=x^2-2x-5$

93. Find the x-intercepts of the graph of the equation.
$y=x^2-5x+6$

94. What is one fact about an even function?

95. What is one fact about an odd function?

96. Use the graph to estimate the roots of the equation.



Formulas to memorize

- Pieces of information, know all

☆Equivalent formulas, choose either formula

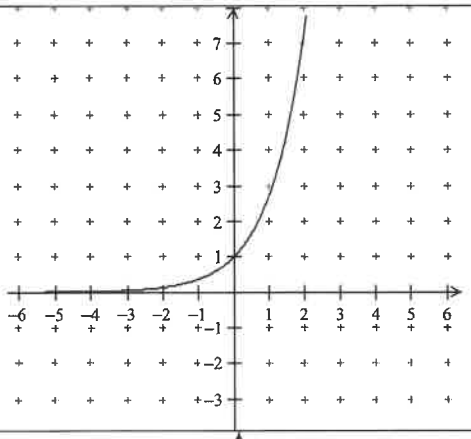
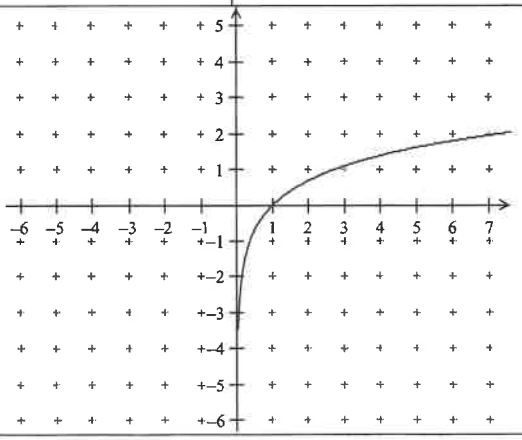
Calc 2 only

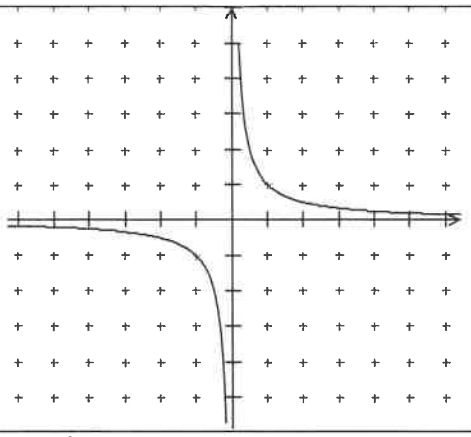
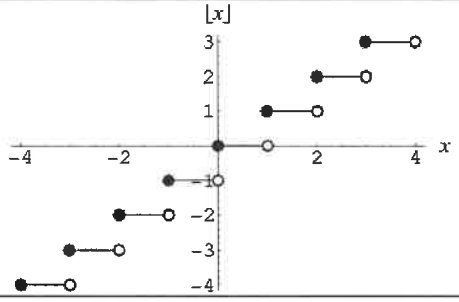
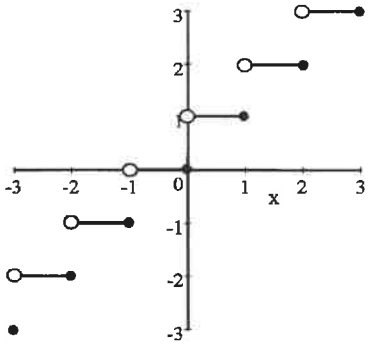
* Won't ask unprompted

1.		Quadratic formula	$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$
2.		$(a+b)^2=$	$a^2+2ab+b^2$
3.		$(a-b)^2=$	$a^2-2ab+b^2$
4.		$(a+b)(a-b)=$	a^2-b^2
5.		$(a+b)^3$	$a^3+3a^2b+3ab^2+b^3$
6.		$a^2-b^2=$	$(a+b)(a-b)$
7.		$a^3+b^3=$	$(a+b)(a^2-ab+b^2)$
8.		$a^3-b^3=$	$(a-b)(a^2+ab+b^2)$
9.		Distance formula	$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
10.		Point-slope form of a line	• $y-y_1=m(x-x_1)$ • m=slope • (x_1,y_1) is the point
11.		Slope-intercept form of a line	• $y=mx+b$ • m=slope • b = y-intercept
12.		Circle formula	• $(x-h)^2+(y-k)^2=r^2$ • Center (h,k) • r = radius
13.		Standard form of a quadratic	• $y=ax^2+bx+c$ • a determines if it opens up or down • c is the y-intercept • Vertex at $\left(\dfrac{-b}{2a},f\left(\dfrac{-b}{2a}\right)\right)$
14.		Vertex form of a quadratic	• $y=a(x-h)^2+k$ • Vertex: (h,k) • Axis of Symmetry (AoS): $x=h$ • Extreme value = k
15.		Intercept form of a quadratic	• $y=a(x-r)(x-s)$

			• X-intercepts at r and s • Vertex is halfway between r and s
16.		Parabola formula (conic)	• $y=\frac{1}{4c}(x-h)^2+k$ or $\sqrt{17}+5\sqrt{17}$ • vertex (h,k) • focus = vertex + c on variable that is not squared; like $(h,k+c)$ • directrix: equation with vertex – c on variable that is not squared; like $y=k-c$
17.		Ellipse formula	• $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ • $c^2=a^2-b^2$ where a is the larger radius • Center (h,k) • x-radius a • y-radius b • foci c units from (h,k) along the major axis
18.		Hyperbola formula	• $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ • $c^2=a^2+b^2$ • Center (h,k) • x-radius a • y-radius b • foci c units from (h,k) along the first axis
19.	*	eccentricity	$e=\frac{c}{a}$
20.		Area of a circle	$A=\pi r^2$
21.		Area of a trapezoid	$A=\frac{1}{2}(b_1+b_2)h$
22.		Volume of a Cube	s^3
23.		Volume of a cone or pyramid	$\frac{1}{3}A_{base}h$
24.		Volume of a sphere	$\frac{4}{3}\pi r^3$
25.		Compounding interest	• $A=P\left(1+\frac{r}{n}\right)^{t\cdot n}$ • $P=$ Principal • $r=$ interest rate (APR), as a decimal • $n=$ number of times interest is compounded in a year • $t=$ time in years

26.	*	Definition of e	As n approaches $+\infty$, $\left(1+\frac{1}{n}\right)^n$
27.		continuous compound interest	☆ $A(t)=A_o e^{rt}$ ☆ $A(t)=Pe^{rt}$
28.		$\sin^2 x+\cos^2 x=$	1
29.		$1+\tan^2 x=$	$\sec^2 x$
30.		$1+\cot^2 x=$	$\csc^2 x$
31.		$\sin(A\pm B)=$	$\sin A\cdot\cos B\pm\cos A\cdot\sin B$
32.		$\cos(A\pm B)=$	$\cos A\cdot\cos B\mp\sin A\cdot\sin B$
33.		$\tan(A\pm B)=$	$\frac{\tan A\pm\tan B}{1\mp\tan A\cdot\tan B}$
34.		$\sin(2u)=$	$2\sin u\cdot\cos u$
35.		$\cos(2u)=$	$\cos^2 u-\sin^2 u$
36.		$\tan(2u)=$	$\frac{2\tan u}{1-\tan^2 u}$
37.		$\sin^2 u=$	$\frac{1-\cos 2u}{2}$
38.		$\cos^2 u=$	$\frac{1+\cos 2u}{2}$
39.		$\tan^2 u=$	$\frac{1-\cos 2u}{1+\cos 2u}$
40.	*	$\sin A\cdot\sin B=$	$\frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$
41.	*	$\cos A\cdot\cos B=$	$\frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$
42.	*	$\sin A\cdot\cos B=$	$\frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right]$
43.	*	$\cos A\cdot\sin B=$	$\frac{1}{2}\left[\sin(A+B)-\sin(A-B)\right]$
44.	*	$\cot^{-1}(x)=$	$\frac{\pi}{2}-\tan^{-1}(x)$
45.	*	$\sec^{-1}(x)=$	$\cos^{-1}\left(\frac{1}{x}\right)$
46.	*	$\csc^{-1}(x)=$	$\sin^{-1}\left(\frac{1}{x}\right)$
47.		general equation for a sinusoid	• $y=D+A\cos\left[B(x-C)\right]$ • sinusoidal axis is $y=D$ • amplitude $=\lvert A\rvert$

			• period $= \dfrac{2\pi}{\lvert B \rvert}$ • C is horizontal translation/phase shift
48.		law of sines	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
49.		law of cosines	$c^2 = a^2 + b^2 - 2ab\cos C$
50.		area of triangle using trig	Area $= \dfrac{1}{2}ab\sin C$
51.	#	parameterization of ellipse	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ becomes $x = a\cos t$, $y = b\sin t$
52.		$\log_b\left(c^d\right) =$	$d\log_b(c)$
53.		$\log_b(cd) =$	$\log_b(c) + \log_b(d)$
54.		$\log_b\left(\dfrac{c}{d}\right) =$	$\log_b(c) - \log_b(d)$
55.		Change of base rule for logs	$\log_a x = \dfrac{\ln x}{\ln a}$
56.		$\ln 1 =$	0
57.		$e^{\ln x} =$	x
58.		$\ln e^x =$	x
59.		$y = e^x$ (graph)	
60.		$y = \ln x$ (graph)	

61.		$f(x)=\frac{1}{x}$ (graph)	
62.		floor function (def)	☆ Greatest integer that is less than or equal to x. ☆ Round down to the nearest integer
63.		$\lfloor x \rfloor = \text{int}(x) = [x]$ graph	
64.		ceiling function (def)	☆ Least integer that is greater than or equal to x. ☆ Round up to the nearest integer
65.		$\lceil x \rceil$ (graph)	
66.		$\lvert x \rvert =$ (as a piecewise function)	$\begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$